HOMEWORK 3 EXTRA EXERCISE 3 SOLUTION

Exercise 1. Compute the following coefficients. You *do not need* to expand the entire expression.
(a) What is the coefficient of a^4b^3 in $(a+b)^7$?
(b) What is the coefficient of x^4y^6 in $(2y^3+5x^2)^4$?

Solution The binomial expansion is in general,

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

This is what we will use to find the sought after coefficients.

(a) The coefficient of a^4b^3 in the expansion of $(a+b)^7$ is $\binom{7}{3}$. It accounts for the number of ways we can pick 4 copies of a out of 7 possible copies of a when we consider the product

$$(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)(a+b).$$

Note that equivalently we can count the number of ways of picking 3 out of 7 copies of b to combine, but

$$\binom{7}{3} = \binom{7}{4},$$

that's an equivalent way of getting the answer.

Now

$$\binom{7}{3} = \frac{7!}{3!4!} = 5 \cdot 7 = 35,$$

so the coefficient of a^4b^3 in the expansion of $(a+b)^7$ is 35.

(b) The coefficient of $(2y^3)^k(5x^2)^{4-k}$ in the expansion of $(2y^3+5x^2)^4$ is $\binom{4}{k}$, for any $0 \leq k \leq 4$. We are looking specifically for the coefficient of x^4y^6, and that is the term in the expansion corresponding to $k=2$. The term in the expansion looks like

$$\binom{4}{2}(2y^3)^2(5x^2)^2 = (\frac{4!}{2!2!} \cdot 4 \cdot 25)\ x^4y^6 = 600x^4y^6.$$

So the coefficient of x^4y^6 is 600.